Special Theory of Relativity

(Einstein postulates And Lorentz Transformations)

e-content for B.Sc Physics (Honours)
B.Sc Part-I

Paper-I

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Special Theory of Relativity

Special ⟹ means that the theory applies only to ***inertial*** *reference frames*.

Theory ⟹ means that the concept has been confirmed by many different experiments.

Relativity ⟹ means there is **no absolute** frame of reference and hence, any measured values must be **relative.**

Einstein's postulates

Einstein based his *special theory of relativity* on two fundamental postulates:

❶ The principle of relativity:

Gone with the stationary ***ether*** was the idea of ***an absolute*** frame of reference. All motion is relative, not to any stationary system in the universe, but to selected frames of reference.

Thus, for a passenger on a train with no windows, there would be no way to determine whether the train is moving with uniform velocity or is at rest.

This is the first of Einstein's postulates:

All laws of physics have the same mathematical form in all inertial reference frames.

This means:

- There is no preferred frame of reference.
- There is no a physical experiment, *mechanical, electrical or optical* can be performed to determine our state of uniform motion.
- Galilean transformations are not correct for all laws of physics.

❷ The constancy of the speed of light:

Q- "What would a light beam look like if you traveled along beside it?"

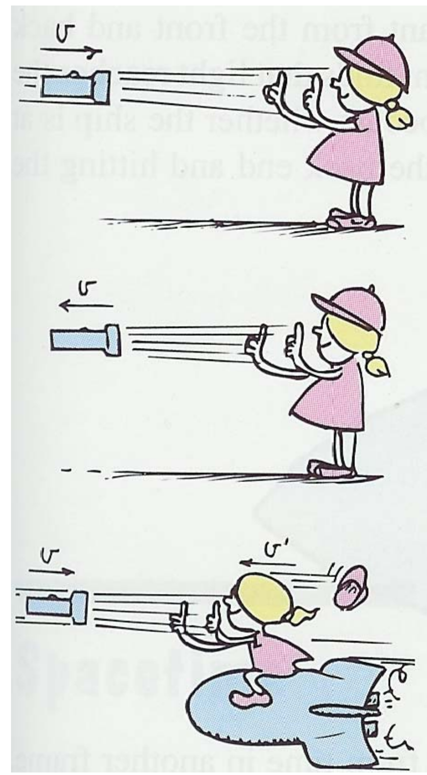
A- According to classical physics, the beam would be **at rest** to such an observer.

The more Einstein thought about this, the more convinced he became that one could not move with a light beam. He finally came to the conclusion that no matter how close a person comes to the speed of light, he would still measure the light at $c = 3\times10^8$ **m/s**.

This was the second postulate in his special theory of relativity:

The speed of light in a vacuum has the same measured value (c) in all inertial reference frames.

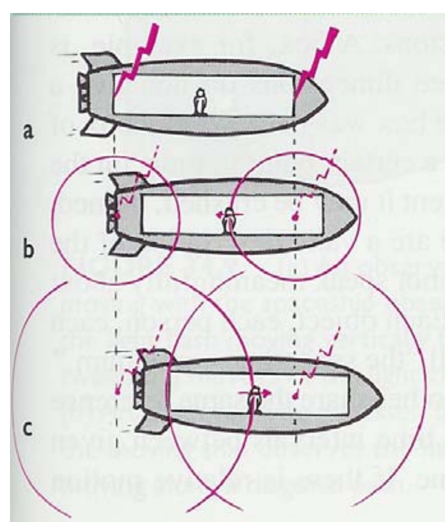
This means:



- The speed of light is invariant.
- The classical idea that space & time are **independent** had to be rejected. (I.e. there is should be a relationship between space & time).
- As a consequence of Einstein's 2^{nd} postulate , is *the concept of Non-Simultaneity*.

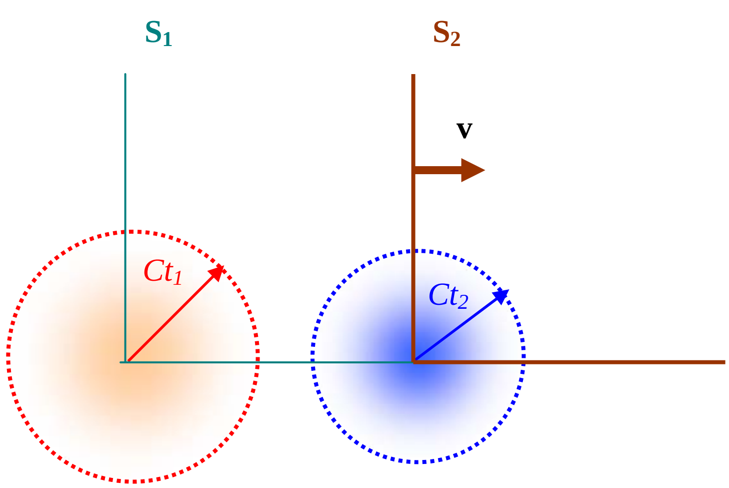
This concept states that:

Two events that are simultaneous in one frame of reference ***need not be*** *simultaneous in a frame moving relative to the first frame.*



Lorentz Transformations

- This transformation derives its name from the Dutch physicist **Hendrik Lorentz** (1853-1928).
- Unlike Galilean transformations, Lorentz transformations involve a change of *spatial distance* and a change of *time interval* between two inertial systems. I.e. they are **space-time** transformations.
- Suppose that the coordinate system S_2 is moving with constant velocity v along the x-axis of the coordinate system S_1 , where $y_2 = y_1$ and $z_2 = z_1$.



- Suppose that at $t_1 = t_2 = 0$ a point source of light at the common origin sends out a spherical pulse of light.

- Since c is **a constant** for all observers in both S_1 and S_2 and is the same in all directions, all observers in both frames of reference must detect a spherical wavefront expanding from their origin.

- Since the equation of a sphere is $x^2 + y^2 + z^2 = r^2$ and r, the radius, equals ct, we can write

$$x_1^2 + y_1^2 + z_1^2 - c^2 t_1^2 = 0$$

$$x_2^2 + y_2^2 + z_2^2 - c^2 t_2^2 = 0$$

- It is easy to see that Galilean transformations will not satisfy both these equations (recall : $x_1 = x_2 + vt$).

- Lorentz derived some formulas that can satisfy both above equations. Those formulas known as ***Lorentz transformations***, and they are:

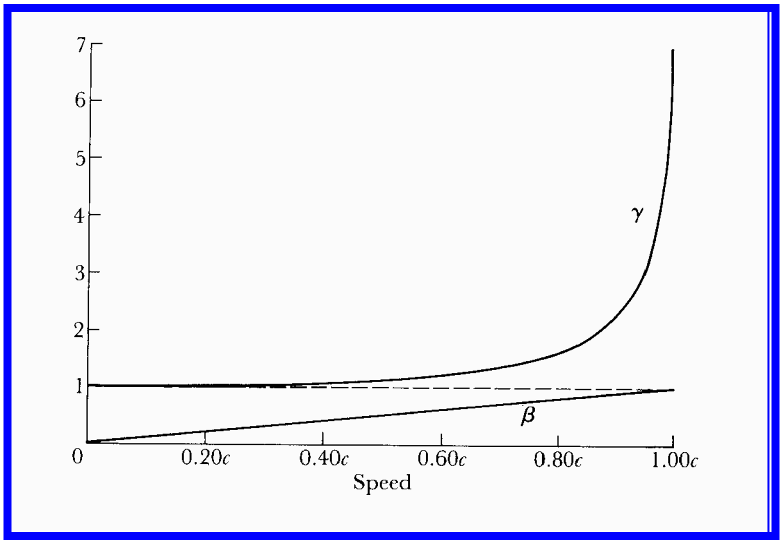
$$
\left.\begin{aligned}
x_2 &= \gamma(x_1 - vt_1) \\
y_2 &= y_1 \\
z_2 &= z_1 \\
t_2 &= \gamma\left(t_1 - \frac{v}{c^2}x_1\right)
\end{aligned}\right\}
$$

where

$$
\gamma = \frac{1}{\sqrt{1-\beta^2}}
$$

$$
\beta = \frac{v}{c}
$$

Note that γ is always greater than or equal to 1 for β less than or equal to 1.



The Correspondence Principle

The *correspondence principle* states that:

Any new theory in physics must reduce to its corresponding well-established classical theory in the situations for which the classical theory is valid.

Or, simply:

new theory + old one must correspond.

Since the physics of Galileo and Newton was experimentally established for objects that moved at speeds much less than the speed of light. We **should** then **find** that the relativistic Lorentz transformations reduce to the classical Galilean transformations as v/c $=\beta$ approaches zero. Applying such case into Lorentz transformations shows that as:

$$\beta \Rightarrow 0, \gamma \Rightarrow 1$$

All transformations reduce to the classical Galilean ones.

*Therefore, Lorentz transformations **do** indeed **agree** with the correspondence principle.*